

CHAPTER TWO

Describing test scores

II. Conceptual exercises

A. Matching

1. g
2. i
3. a
4. b
5. e
6. c
7. f
8. d

B. True or false

1. T
2. T
3. T
4. T
5. F
6. T
7. T
8. F
9. T
10. T
11. F
12. T

C. Brief responses

1. The distribution is skewed, either negatively or positively.
2. The distribution is negatively skewed. A typical score distribution of a school-level class where the students are high achievers is an example.
3. While the means for the two classes are quite close, the standard deviation of Class B is twice that of Class A. What these two pieces of information inform us about the classes is that while the abilities of the students on average are quite similar (judging from the mean), there is wide variation in abilities of the students in Class B when compared to Class A (judging from the standard deviation).

III. Calculations with small data sets

1. Data set 1:

Mode	30
Median	30
Mean	32.40
Range	46

2. Data set 2

Mode	10, 64
Median	28
Mean	39.20
Range	72

3. Calculate the standard deviation.

Data set 1: 10, 14, 18, 24, 30, 30, 40, 50, 52, 56

X	f	fX	x	fx	x ²	fx ²	X ²	fX ²
10	1	10	-22.4	-22.4	501.76	501.76	100	100
14	1	14	-18.4	-18.4	338.56	338.56	196	196
18	1	18	-14.4	-14.4	207.36	207.36	324	324
24	1	24	-8.4	-8.4	70.56	70.56	576	576
30	2	60	-2.4	-4.8	5.76	11.52	900	1800
40	1	40	7.6	7.6	57.76	57.76	1600	1600
50	1	50	17.6	17.6	309.76	309.76	2500	2500
52	1	52	19.6	19.6	384.16	384.16	2704	2704
56	1	56	23.6	23.6	556.96	556.96	3136	3136
Σ	10	324	-2.4	0.0	2432.64	2438.40	12036	12936

Mean: 32.4

Standard deviation (through SPSS): 16.46

Standard deviation (using Equation 2.7): 15.62

Standard deviation (using Equation 2.8): 15.62

Standard deviation (using Equation 2.10): 16.46¹

Using Equation 2.7:

$$\begin{aligned}
 S &= \sqrt{\frac{\sum fx^2}{\sum f}} \\
 &= \sqrt{\frac{2438.4}{10}} = \sqrt{243.84} = 15.62
 \end{aligned}$$

OR

Using Equation 2.8

$$S = \sqrt{\frac{\sum X^2 - \frac{(\sum X)^2}{N}}{N}}$$

¹ See Chapter 2 of the textbook for why Equation 2.10 produces slightly different results when compared to Equations 2.7 and 2.8.

$$N = 10$$

$$(\Sigma fX)^2 = (324)^2 = 104976$$

$$\Sigma fX^2 = 12936$$

$$S = \sqrt{\frac{12936 - \frac{104976}{10}}{10}}$$

$$S = \sqrt{\frac{12936 - 10497.6}{10}}$$

$$S = \sqrt{\frac{2438.4}{10}}$$

$$S = \sqrt{243.84}$$

$$S = 15.62$$

OR

Using Equation 2.10

$$s = \sqrt{\frac{\Sigma x^2}{N-1}} \text{ for ungrouped scores}$$

Or

$$s = \sqrt{\frac{\Sigma fx^2}{N-1}} \text{ for grouped scores}$$

$$s = \sqrt{\frac{2438.40}{9}}$$

$$S = \sqrt{270.93}$$

$$S = 16.46$$

Data set 2: 10, 18, 36, 64, 20, 82, 12, 10, 64, 76

X	f	fX	x	fx	x²	fx²	X²	fX²
10	2	20	-29.2	-58.4	852.64	1705.28	100	200
12	1	12	-27.2	-27.2	739.84	739.84	144	144
18	1	18	-21.2	-21.2	449.44	449.44	324	324
20	1	20	-19.2	-19.2	368.64	368.64	400	400
36	1	36	-3.2	-3.2	10.24	10.24	1296	1296
64	2	128	24.8	49.6	615.04	1230.08	4096	8192
76	1	76	36.8	36.8	1354.24	1354.24	5776	5776
82	1	82	42.8	42.8	1831.84	1831.84	6724	6724
Σ	10	392	+4.4	0.0	6221.92	7689.60	18860	23056

Mean: 39.2

Standard deviation (through SPSS): 29.23

Standard deviation (using Equation 2.7): 27.73

Standard deviation (using Equation 2.8): 27.73

Standard deviation (using Equation 2.10): 29.23

Using Equation 2.7:

$$S = \sqrt{\frac{\sum fx^2}{\sum f}}$$

$$= \sqrt{\frac{7689.60}{10}} = \sqrt{768.96} = 27.73$$

OR

Using Equation 2.8:

$$S = \sqrt{\frac{\sum X^2 - \frac{(\sum X)^2}{N}}{N}}$$

$$N = 10$$

$$(\Sigma fX)^2 = (392)^2 = 153664$$

$$\Sigma fX^2 = 23056$$

$$S = \sqrt{\frac{23056 - \frac{153664}{10}}{10}}$$

$$S = \sqrt{\frac{23056 - 15366.4}{10}}$$

$$S = \sqrt{\frac{7689.6}{10}}$$

$$S = \sqrt{768.96}$$

$$S = 27.73$$

OR

Using Equation 2.10:

$$s = \sqrt{\frac{\Sigma x^2}{N-1}} \text{ for ungrouped scores}$$

Or

$$s = \sqrt{\frac{\Sigma fx^2}{N-1}} \text{ for grouped scores}$$

$$s = \sqrt{\frac{7689.60}{9}}$$

$$S = \sqrt{854.40}$$

$$S = 29.23$$

4. Difference between the two score distributions:

Data set 1 is more homogenous when compared to the data set 2. This can be checked with a histogram or kurtosis and skewness for the data sets. [Data set 2 has two modes, while data set 1 has only one; on average scores in data set 2 are higher than those in data set 1 as shown in the means but the score variability in data set 2 is greater as shown in the standard deviations; the relatively large distances among the mean, mode and median for data set 2 indicate skewness of the distribution.]

5. Calculate the descriptive statistics

Data set 3: 29, 33, 32, 36, 34, 32, 17, 35, 31, 36, 37, 40, 32, 31, 24.

Statistic	Values
N	15
Mode	32
Median	32
Mean	32
Range	23
Standard deviation	5.38

Shape of the score distribution:

The values of central tendency (mode, median and mean) presented above show that the data set is normally distributed.

VII. SPSS exercises

1. Descriptive statistics from the FREQUENCIES procedure

Data set 'CTCS.sav'

a. Descriptive statistics

	Tlcstd	Tswstd	Tvrstd
Valid N	1448	1448	1448
Missing N	40	40	40
Mean	49.62	51.12	51.49
Median	50	51	52
Mode	51	54	54
Std. deviation	6.67	6.90	6.70
Variance	44.43	47.60	44.84
Skewness	-.074	-.241	-.503
Std. error of skewness	.064	.064	.064
Kurtosis	-.293	-.041	.080
Std. error of kurtosis	.129	.129	.129
Range	39	43	38
Minimum	29	25	28
Maximum	68	68	66
Sum	71848	74019	74556

b. Shapes of the score distributions:

Tlcstd: The shape of the score distribution is bell-shaped. The skewness value is close to zero and the kurtosis value is within the normal range.

Tswstd: The shape of the score distribution is not as bell-shaped as in "Tlcstd"; there is some skewness (notice the distribution is a little off-center) but the shape is not as peaked as kurtosis is nearer to zero than in Tlcstd.

Tvrstd: The shape of the score distribution is less bell-shaped than even 'Tswstd'; there is more skewness here (notice the distribution is quite off-center) and the skewness is more than in other two variables. Kurtosis is within the normal range.

2. Descriptive statistics and histograms from the FREQUENCIES procedure
Data set 'CTCS.sav'

a. Descriptive statistics

	Tstdtot	Spgram	Sppron
Valid N	1448	1304	1314
Missing N	40	184	174
Mean	507.43	1.93	2.13
Median	510	1.90	2.20
Mode	497*	2	2
Std. deviation	58.86	.45	.38
Variance	3464.87	.21	.14
Skewness	-.280	.010	-.919
Std. error of skewness	.064	.068	.067
Kurtosis	-.233	-.115	1.731
Std. error of kurtosis	.129	.135	.135
Range	337	3	3
Minimum	310	0	0
Maximum	647	3	3
Sum	734755	2521	2804

b. Shapes of the score distributions:

Tstdtot: The shape of the score distribution is quite bell-shaped although there is a little skewness (notice the distribution is a little bit off-center) and the shape is a little flat.

Spgram: The shape of the score distribution is quite bell-shaped; there is very little skewness and very little peakedness.

Sppron: The shape of the score distribution is not bell-shaped; the distribution is very skewed (notice the distribution is quite off-center) and is slightly peaked.

3. Interpreting histograms

(i) FCE Paper 1—The shape of the score distribution is quite bell-shaped although there is a little negative skewness (notice the distribution is a bit off-center to the right).

(ii) FCE Paper 2—The shape of the score distribution is not bell-shaped; the distribution is slightly negatively skewed (notice the distribution is a little off-center to the right) but the distribution is quite peaked.

(iii) FCE Paper 4: Grade—The shape of the score distribution is not bell-shaped; the distribution is positively skewed (notice the distribution is quite off-center to the left).

(iv) FCE Paper 5: Fluency rating—The shape of the score distribution is not bell-shaped as it is slightly negatively skewed (notice the distribution is a bit off-center to the right) and the distribution is also very peaked.